C2 Jan 2013 (MA)

Q1) $(2-5x)^6 \simeq (2)^6 + \binom{6}{1}(2)^5(-5x)^1 + \binom{6}{2}(2)^4(-5x)^2 + \ldots$

$\simeq 64 - 192(5x) + 240(25x^2) + \ldots$

$\simeq \boxed{64 - 960x + 6000x^2}$

Q2a) $f(1) = 0$: $f(1) = a + b - 4 - 3 = 0$

$\therefore a + b = 7$

b) $f(-2) = 9$: $a(-2)^3 + b(-2)^2 - 4(-2) - 3 = 9$

$-8a + 4b + 5 = 9$

$4b - 8a = 4$

$\therefore b - 2a = 1$

$[b + a = 7]$

$-\ [b - 2a = 1]$

$0 + 3a = 6$

$\therefore a = \frac{6}{3} = \boxed{2}$ and $b = 7 - 2 = \boxed{5}$

Q3a)

a	ar	ar^2	ar^3
2013	2014	2015	2016

$a = 120000$

$r = 1.05$

$ar^3 = 120000 \times (1.05)^3 = \boxed{£138915}$

b) $ar^{N-1} > 200000$

$r^{N-1} > \frac{200000}{a}$

$\log(r^{N-1}) > \log\left(\frac{200000}{a}\right)$

$(N-1)\log 1.05 > \log\left(\frac{200000}{120000}\right)$

$(N-1) > \frac{\log\left(\frac{5}{3}\right)}{\log(1.05)}$ $\therefore N-1 > 10.47\ldots$

$N > 11.47\ldots$

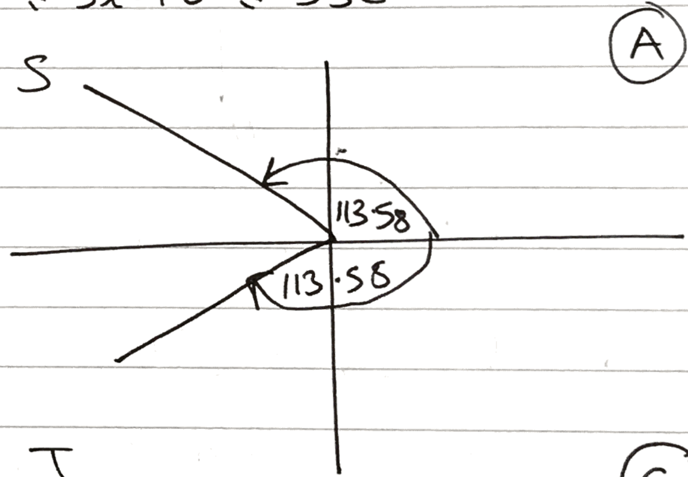
so 12^{th} term $\longrightarrow$ $\boxed{2024}$

c) $S_{11} = \frac{120000(1-(1.05)^{11})}{1-1.05} = \boxed{£1704814}$

4) $\cos(3x-10) = -0.4$

$\cos^{-1}(-0.4) = 3x-10 = 113.58°$

<u>solving in:</u> $-10° \leq 3x-10 \leq 530°$



$3x-10 = 113.58°, (360-113.58°), (360+113.58°)$

$3x-10 = 113.58°, 246.42°, 473.58°,$

$3x = 123.58°, 256.42°, 483.58°$

$\boxed{x = 41.2°, 85.5°, 161.2°}$

5a i) completing square

$$(x-10)^2 - 100 + (y-12)^2 - 144 + 195 = 0$$

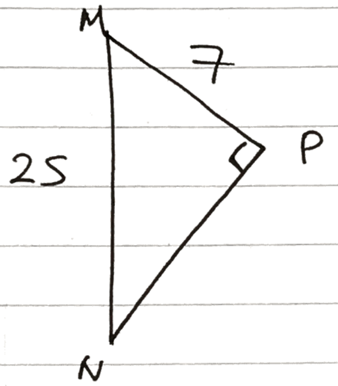
$$(x-10)^2 + (y-12)^2 = 49$$

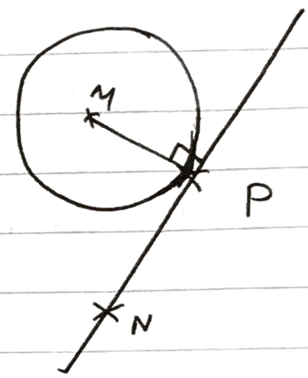
$\therefore$ centre : $(10, 12)$

ii) radius $= \sqrt{49} = 7$

b) $|MN| = \sqrt{(25-10)^2 + (32-12)^2} = \underline{\underline{25}}$ units

c)





$|PN| = \sqrt{25^2 - 7^2} = \boxed{24}$ units

Q6a) $\log_2[(x+15)^2] - \log_2(x) = 6$

$$\log_2\left(\frac{(x+15)^2}{x}\right) = 6$$

$$2^6 = \frac{(x+15)^2}{x} = 64$$

$$(x+15)^2 = 64x$$

$$x^2 + 30x + 225 = 64x$$

$$x^2 - 34x + 225 = 0$$

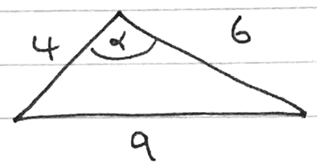
□

b) $x^2 - 34x + 225 = 0$

$(x-9)(x-25) = 0$

$\boxed{x = 9}$, $\boxed{x = 25}$

Q7a)

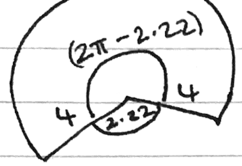


<u>cosine rule</u>

$\cos\alpha = \dfrac{6^2 + 4^2 - 9^2}{2(6)(4)} = \dfrac{-29}{48}$

$\therefore \alpha = \cos^{-1}\left(\dfrac{-29}{48}\right) = \boxed{2.22^c}$

b)



Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(4^2)(2\pi - 2.22) = \boxed{32.5}\ \text{cm}^2$

c) Area $\Delta XZY = \frac{1}{2}(6)(4)(\sin 2.22) = 9.559\ldots$

$\therefore$ total area $= 32.5 + 9.559\ldots = \boxed{42.1\ \text{cm}^2}$

d) $WY = 6 - 4 = 2\text{cm}.$

ZW length $= 4 \times (2\pi - 2.22^c) = 16.253\ldots$

Perimeter $= 16.25 + 2 + 9 = \boxed{27.3\ \text{cm}}$

8a) $\frac{dy}{dx} = -3 + 12x^{-4}$

$\hookrightarrow \underline{x = \sqrt{2}}$: $\frac{dy}{dx} = -3 + \frac{12}{(\sqrt{2})^4} = -3 + 3 = 0$ //

$\therefore$ there is a turning point at $x = \sqrt{2}$.

b) solving $[-3 + 12x^{-4} = 0]$ will yield $x = \sqrt{2}$ and $\boxed{x = -\sqrt{2}}$

c) $\frac{d^2y}{dx^2} = -48x^{-5}$

d) $\underline{x = -\sqrt{2}}$: $\frac{d^2y}{dx^2} > 0$ $\therefore$ minimum at $x = -\sqrt{2}$

$\underline{x = \sqrt{2}}$: $\frac{d^2y}{dx^2} < 0$ $\therefore$ maximum at $x = -\sqrt{2}$

Q9a)

x	2	3
y	6.272	3.634

b) $h = \frac{b-a}{n} = \frac{4-1}{6} = \frac{1}{2}$

Area $\simeq \frac{1}{2} \times \frac{1}{2} [25.866 + 6.272 + 5.21 + 3.634 + 1.856)]$

$\simeq \boxed{11.42}$

c) $R = \int_1^4 (27 - 2x - 9x^{\frac{1}{2}} - 16x^{-2})\,dx = \left[27x - x^2 - 6x^{\frac{3}{2}} + 16x^{-1}\right]_1^4$

$= [48] - [36] = \boxed{12}$ units^2